Collections of mathematical objects

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It is a truth almost universally acknowledged, that a discipline in possession of elegant theorems must be in want of nothing so vulgar as data.

1. An overview

* The rest of this talk (left)

2. What is data (in math)?
   Terminology, history and the present

3. Where is it & what is it like?
   Towards a survey

4. What can we do better?
   Data management, a TL;DR

5. New(ish) uses?
   Validation and correctness

Data Management Plan

No data management plan is necessary, since the research outlined in this proposal is in the realm of Mathematics and by nature theoretical. The PI will make his research freely and publicly accessible through articles, graphics and programs on the web page <INSERT WEBSITE>. The PI will also submit articles and papers to appropriate peer-reviewed journals for publication. Finally this work will be disseminated through academic research seminars and conferences.

www.math.harvard.edu/media/DataManagement.pdf
(link curtesy of Boege et al)

2. Research data

Collections of examples and beyond

- "Research data are all digital and analog objects generated or handled in the process of doing research" *(MaRDI)*
- *right*
  The listing on the Cornell Mathematics Library page on "Math Databases".
- Today: collections of examples.

- MathSciNet
- Zentralblatt fur Mathematik
- Google Scholar
- Wikipedia
- MacTutor History of Mathematics
- Scopus
- The Web of Science
- Mathworld
- Jahrbuch-Project Electronic Research Archive for Mathematics (mathematics literature 1868 - 1943)
- arXiv
- ERIC (index in the field of Education, including Education in Mathematics)
- Wolfram|Alpha ("this search engine allows you to enter a query and returns an answer from structured data")

"All digital and analog objects" includes: paper publications, proofs, computational results (and more).

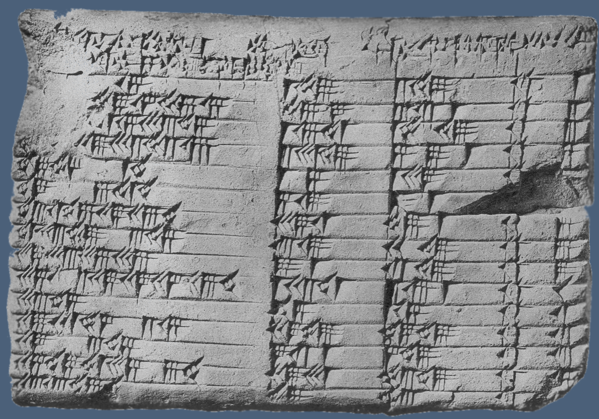
Data outlasting the computation

Persistence before computers

- *Earliest datable:* length measurements and corresponding areas. (*Sumer, c. 2600 BCE)*

- *picture:* Pythagorean triples (Babylon, c 1800 BCE).

- *c. 417 - c. 369 BCE:* Platonic solids *(Theaetetus of Athens).*



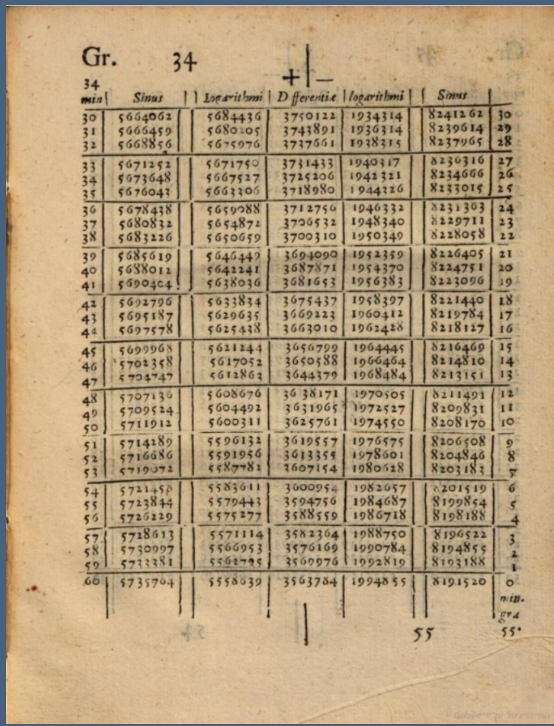
Bill Casselman

Eleanor Robson, "Tables and tabular formatting in Sumer, Babylonia, and Assyria, 2500 BCE-50" The History of Mathematical Tables from Sumer to Spreadsheets [2003] Campbell-Kelly et al [eds].

Data outlasting the computation

Persistence before computers

- Trigonometric, logarithmic, and exponential functions become subjects of tables.
- *picture:*
  Napier, *Mirifici logarithmorum*, trig and log trig data for 34°



Gr. 34

\+ | −

34 min	Sinus	Logarithmi	Differentiae	Logarithmi	Sinus	
30	5664062	5684436	3750122	1934314	8241262	30
31	5666459	5680205	3743891	1936314	8239614	29
32	5668856	5675976	3737661	1938315	8237965	28
33	5671252	5671750	3731433	1940317	8236316	27
34	5673648	5667527	3725206	1942321	8234666	26
35	5676043	5663306	3718980	1944326	8233015	25
36	5678438	5659088	3712756	1946332	8231303	24
37	5680832	5654872	3706532	1948340	8229711	23
38	5683226	5650659	3700310	1950349	8228058	22
39	5685619	5646449	3694090	1952359	8226405	21
40	5688012	5642241	3687871	1954370	8224751	20
41	5690404	5638036	3681653	1956383	8223096	19
42	5692796	5633834	3675437	1958397	8221440	18
43	5695187	5629635	3669223	1960412	8219784	17
44	5697578	5625438	3663010	1962428	8218127	16
45	5699968	5621244	3656799	1964445	8216469	15
46	5702358	5617052	3650588	1966464	8214810	14
47	5704747	5612863	3644379	1968484	8213151	13
48	5707136	5608676	3638171	1970505	8211491	12
49	5709524	5604492	3631965	1972527	8209831	11
50	5711912	5600311	3625761	1974550	8208170	10
51	5714289	5596132	3619557	1976575	8206508	9
52	5716686	5591956	3613355	1978601	8204846	8
53	5719072	5587782	3607154	1980628	8203183	7
54	5721458	5583611	3600954	1982657	8201519	6
55	5723844	5579443	3594756	1984687	8199854	5
56	5726229	5575277	3588559	1986718	8198188	4
57	5728613	5571114	3582364	1988750	8196522	3
58	5730997	5566953	3576169	1990784	8194855	2
59	5733381	5562795	3569976	1992819	8193188	1
60	5735764	5558639	3563784	1994855	8191520	0
						min.
						gra
					55	55°

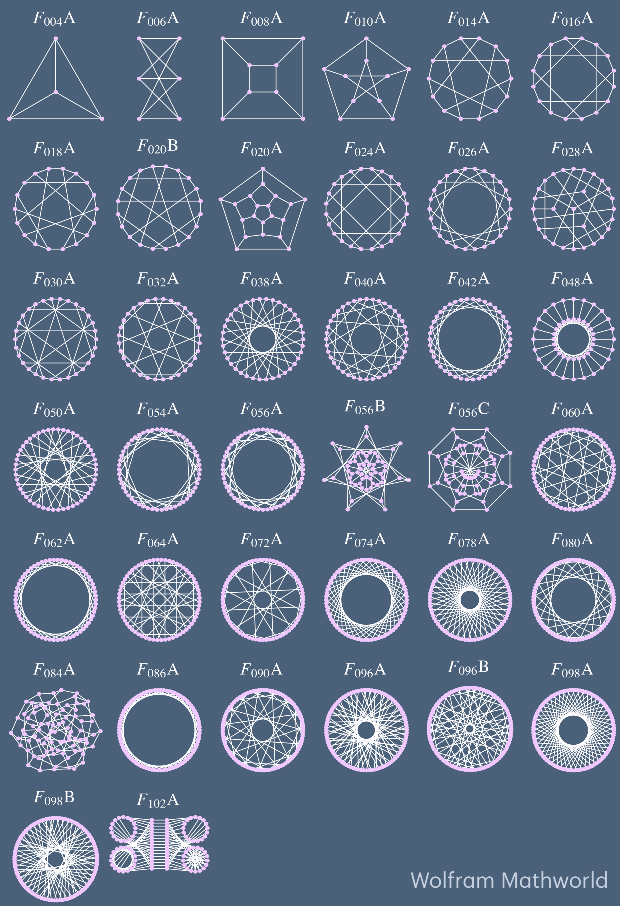
John Napier, 1614

Data outlasting the computation

Persistence before computers

- *1938 - 1946: Math Tables Project (Works Progress Administration)*
  Human computers (450) constructed tables for hand computation.

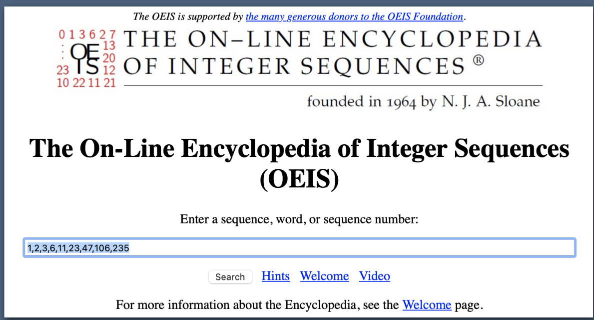
- *picture:*
  The Foster census (1930, published 1988).
  Cubic symmetric graphs with up to 512 vertices. There are 207 such, Foster only missed a handful.

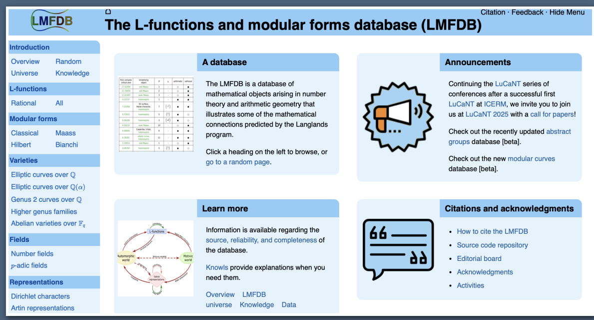


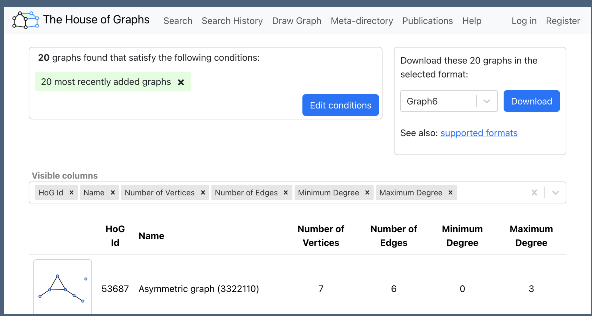
Wolfram Mathworld

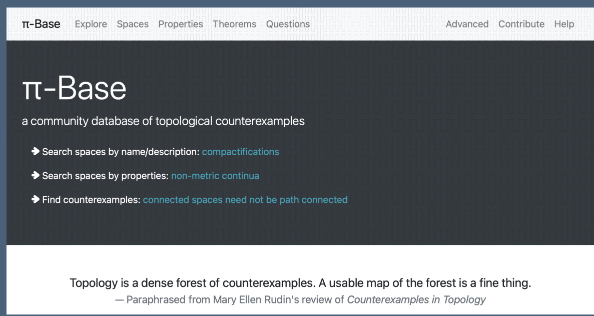
Modern mathematical databases

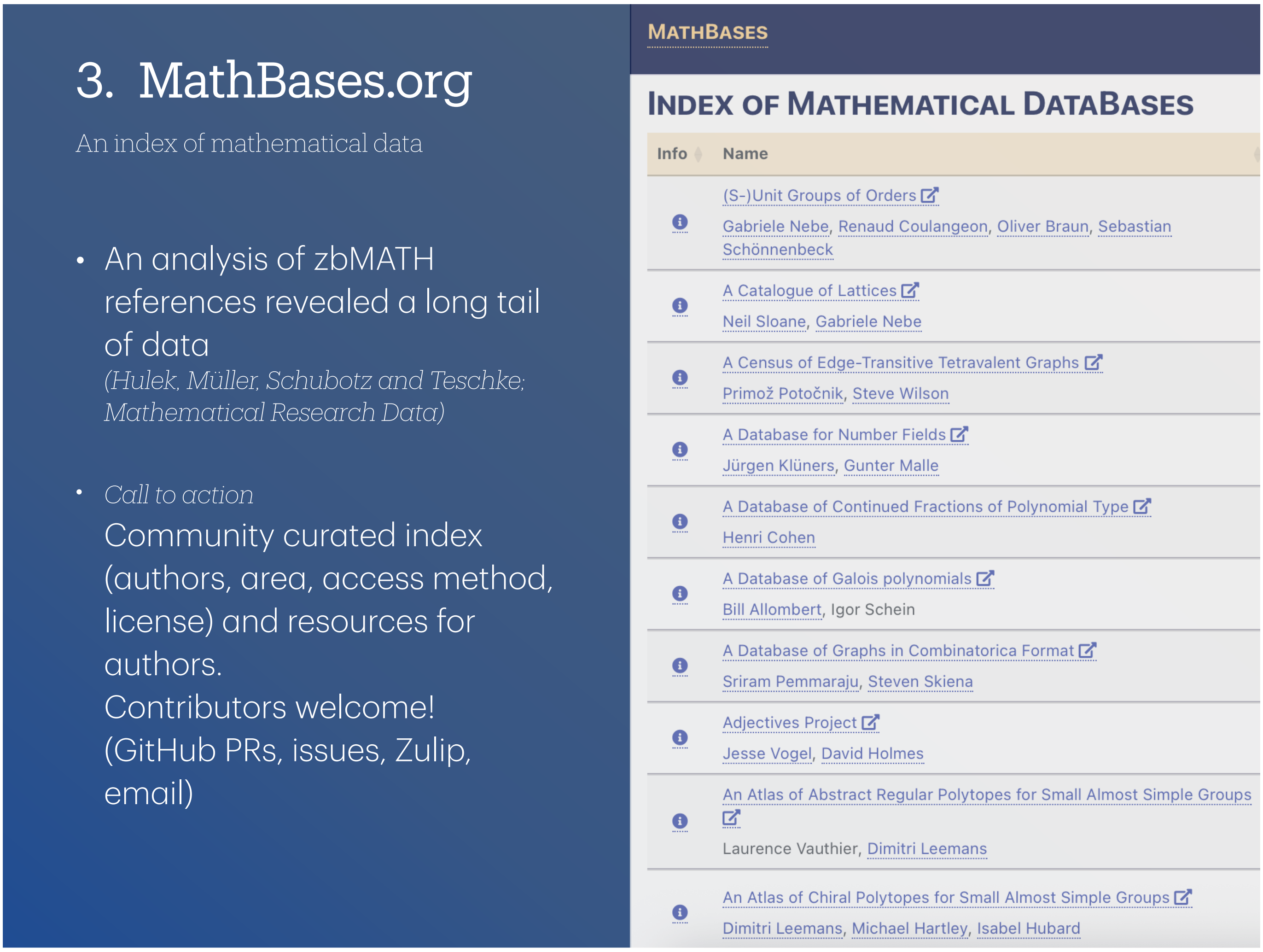
Four well-established examples, just in case









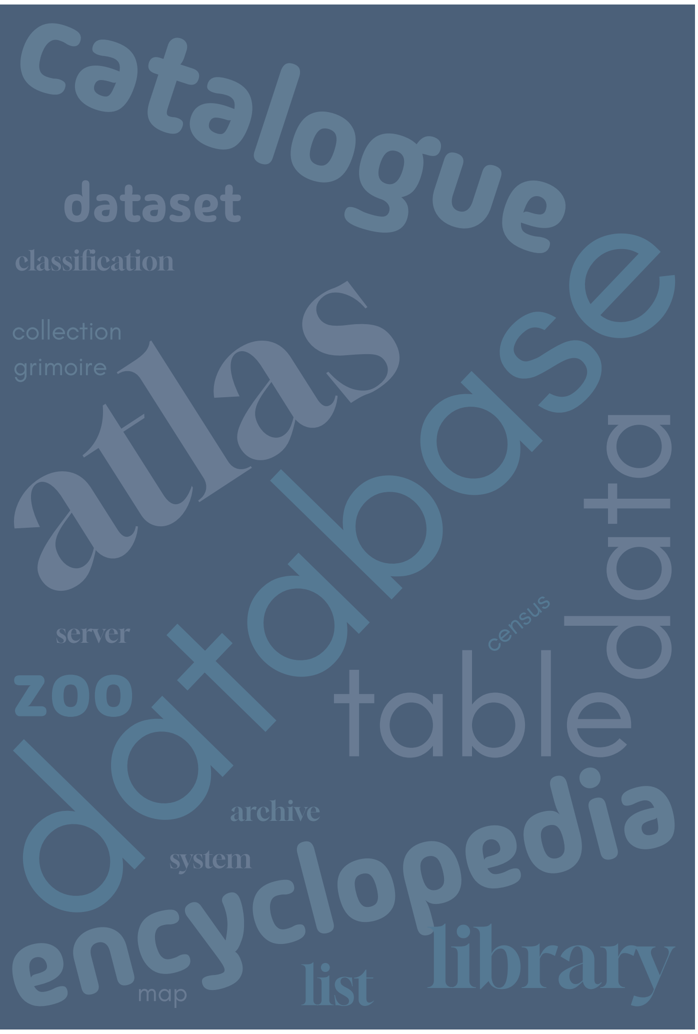


MathBases: index and showcase mathematical databases. 130+ databases indexed so far, with struc-tured metadata: area, access method, license, authors.
A second function: tutorials and templates for people building new databases.

Purposes

Structure reflects intended usage

- Curating (counter)examples
- Index (fingerprint) theorems via simpler objects
- Knowledge reference
- Concept instantiation
- Benchmarking



Curation of examples: topological spaces, properties and theorems in π-base, graphs and invariants in the House of Graphs.
Index theorems: integer sequences (OEIS), Parameters of Strongly Regular Graphs.
Knowledge reference: definitions and properties of special functions in DLMF
Instantiation: datasets in algebraic geometry (only one object, variety)
Benchmarking: SuitSparse matrix dataset

The diversity of collections of examples (databases?)

Four dimensions

small - large

Atlas of Small Chiral Polytopes (56) — *Lists of finite lattices ($17 \cdot 10^9$)*

stored - generated on demand

House of Graphs - The Small Groups Library in GAP - nauty

enumerated - curated/collected

The Foster census — *House of Graphs*

easy to obtain values - a value corresponds to a paper

nauty — *Parameters of Strongly Regular*

4. Doing better

FAIR and RDMP, buzzwords of the day

- Findable, Accessible, Interoperable and Reusable. FAIR ≠ open data
- Research Data Management Plan
- *Call to action*
  What are relevant metadata for collections of examples?

```
List of the 17 representatives of
IC(6,3), ordered by the RevLex-Index.
The numbers above the signs indicate
the elements of the corresponding basis.

                11121121231121231234
                22332334442334445555
                34445555556666666666
IC(6,3, 1) = +++++++++++++-++--++
IC(6,3, 2) = ++++++++++++++++++--
IC(6,3, 3) = +++++++++++++++++++-
IC(6,3, 4) = ++++++++++++++++++++
IC(6,3, 5) = 0++++++++++++++-+---
IC(6,3, 6) = 0+++++++++++++++++--
IC(6,3, 7) = 0+++++++++++++++++++
IC(6,3, 8) = 0++++++++++++++++++0
IC(6,3, 9) = 0++++++0+++++++++---
IC(6,3,10) = 0++++++0++++++++++--
IC(6,3,11) = 0++++++0+++++++++++-
IC(6,3,12) = 0++++++0+++++++0++--
IC(6,3,13) = 0++++++0+++++++0+0--
IC(6,3,14) = 0++++++0++++++0-+---
IC(6,3,15) = 0000++++++++++++++++
IC(6,3,16) = 0000++++++++++++0+++
IC(6,3,17) = 0000000000++++++++++
```

Open in "open access" refers to the removal of financial, legal and technical barriers to data, while accessibility in FAIR refers to the data being retrievable by humans and machines.

The FAIR guidelines

Mostly about metadata

- **Findable:** globally unique, persistent IDs, rich metadata, indexing. *Find & identify.*
- **Accessible:** long term storage, well-defined access conditions. *Repositories.*
- **Interoperable:** FAIR knowledge representation language. *Ready to be combined with other data.*
- **Reusable:** clear usage license, provenance, domain-relevant standards, comprehensive and relevant attributes.

Actionable advice

- More and better metadata and documentation.
- Snapshots in machine readable formats on Zenodo, GitHub, etc.
- Record software info (version), attach code.
- Problem with above: journal publications count, data (mostly) does not.

The FAIR guidelines (2016) are intentionally broad and somewhat vague; they are designed to provide communities with a flexible framework that can be further developed and adapted to specific needs. They focus primarily on metadata — covering aspects such as authorship, provenance, licensing, and descriptions of the dataset's contents.

Accessible: personal webpages are not considered longterm storage.

A taste of an RDMP questionnaire

What we may end up dealing it if the current trends continue

- *Project metadata*:
  title, ID, grant reference, PI names, institution, ...
- *Expected questions*:
  - RDMP author,
  - data types, formats, volume and organization,
  - secure storage and backup,
  - documentation,
  - storage type, ...
- *Often disregarded in mathematics*: license.

Possibly does not apply to mathematics:

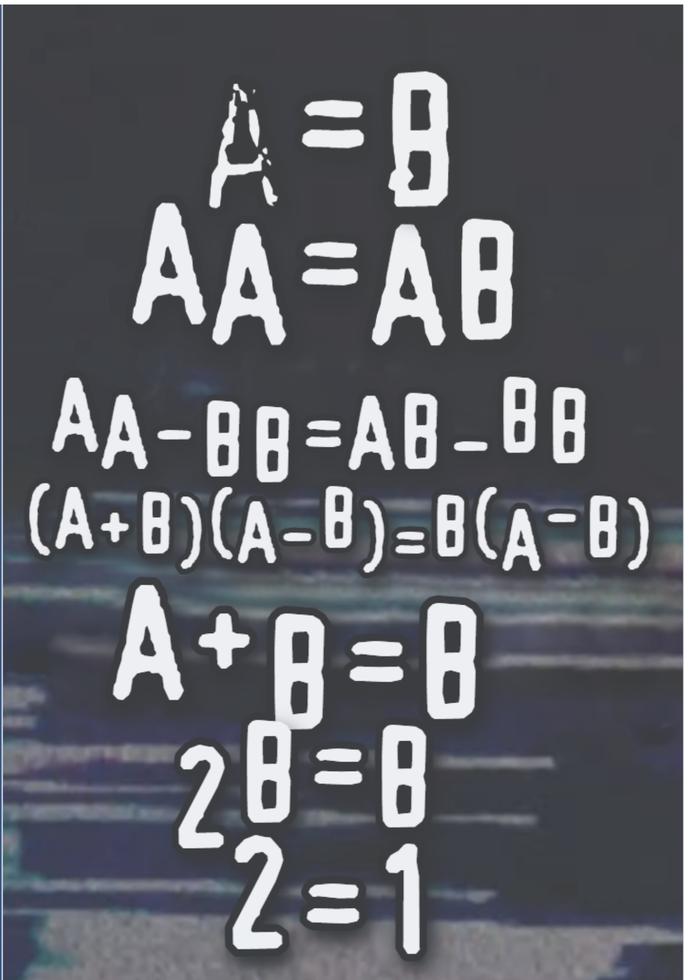
- Ethics approval, legal issues, IP, culturally sensitive issues.
- Data confidentiality and sensitivity, access restrictions (incl. cost)
- Non-digital data questions
- Data destruction

Licensing is complicated by the legal status of mathematical data.
Individual mathematical facts and formulas are generally not copyrightable, but a database as a whole can be protected when the selection or arrangement of its contents constitutes an original intellectual creation; both the European Union's Database Directive and United States copyright law recognize this principle.
A curated collection or a completeness result may therefore carry copyright protection even when the individual entries do not.

5. New(ish) uses

Proof assistants + AI

- Experiments with machine learning on databases of objects.
- Databases as sources of (counter)examples for formalized mathematics.
- Recent developments seem to cause a growing interest in both types of uses.



Curtesy of Steven Clontz

Standard options

And a few deeper questions that we will not answer, just for fun

- **Standard checks**: format, type, consistency, uniqueness, …
- **Testing:** software is run on test cases, the results compared to (trusted) reference results.
- **Redundancy:** several versions of software performing the same task are developed and executed independently, results compared.
- Correctness of code or data established by **formal proofs**.

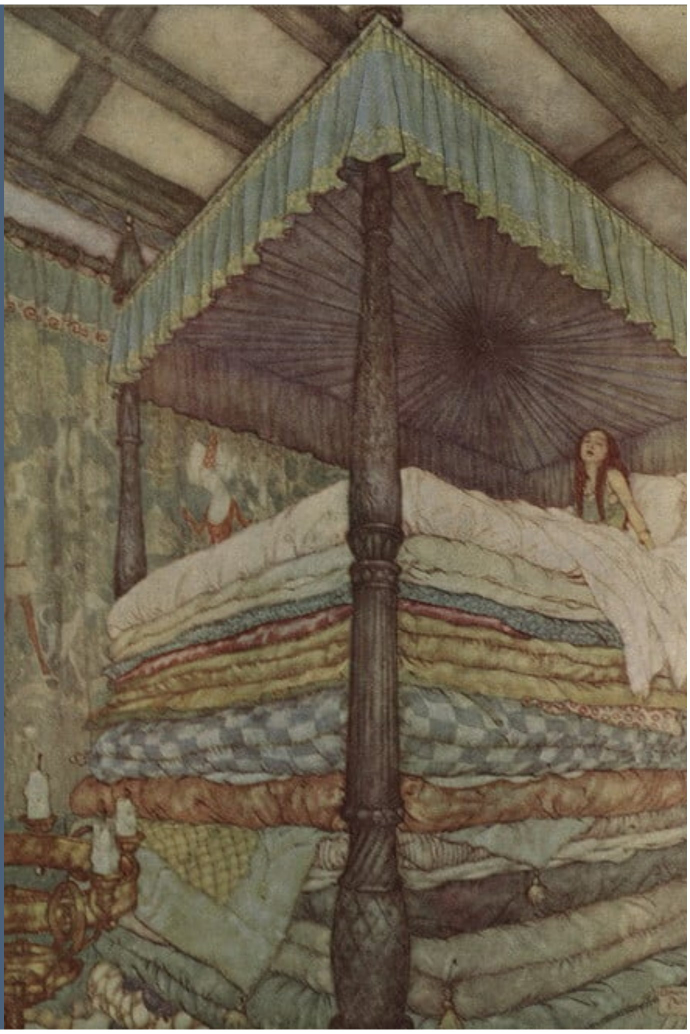
- *How can we trust that a list of examples is complete (if applicable) and correct?*
- *Is the connection between theory and code sound?*
- *Are the results of computations correct?*

No answers to the questions above, just an example: Lean-HoG

Proof assistants

Extremely finicky beasts

- Prove the properties of each example by hand.
- Implement algorithm(s) in the proof assistant.
- Encode as SAT, verify encoding to be correct, use a (trusted) solver, check the solver's certificates.
- Compute properties *and their certificates* with external software, check correctness with a proof assistant.



For few objects and properties, simple.
Few properties, many objects, efficiently computable: can be difficult.
We used a combination of the last two.

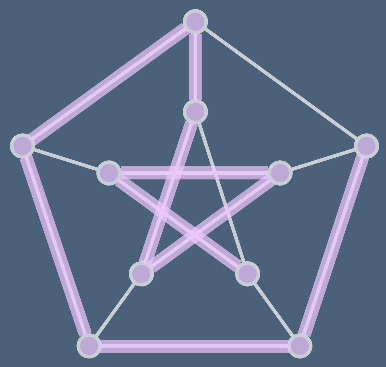
Certificates (a.k.a. witnesses)

Instead of computing values, just check correctness

- Check that 112909084933 is not a prime *vs.* $132241 \cdot 853813 = 112909084933$.

✓ Certificates *explain* why a property holds.

✗ Some properties do not have a certificate.



Easy: find a path in the graph that visits all vertices exactly once.
Hard: prove that we can't find such a cycle in the graph.

Lean-HoG

A Lean 4 library for finite simple graphs incorporating the House of Graphs

- Import efficiently represented graphs into Lean,
- with values and certificates for: the number of connected components, bipartiteness, Hamiltonian paths.
- A tactic to search the database and a tactic to close a goal by finding an example.
- Checking the number of connected components on (almost) all graphs took ~16h.

If you want to try it for yourself:

- Lean may be a sensible proof assistant to start with. A lot depends on Mathlib.
- Checking a database may make you feel like you are doing CS 50 years ago.
- Consider certificates whenever possible.
- Alternative: formal verification of algorithms.

Thank you!

Recap of what we have been up to

- **MathBases**
  Adam Towsley, Ben Spitz, David Roe, David Lowry-Duda, Benjamin Hutz, Edgar Costa, KB
- **Lean-HoG**
  Jure Taslak, Gauvain Devillez, KB, Andrej Bauer
- **MathDataHub**
  Tom Wiesing, KB

Slides (hopefully) and references at katja.not.si, as promised

https://katja.not.si/do-mathematicians-dream-of-data.html