Nut graphs with two vertex and three edge orbits

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Computers in Scientific Discovery

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Joint work with Primož Šparl.

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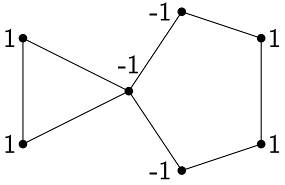

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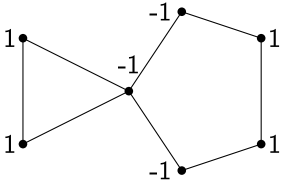

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- Conjecture: Nut graph with $(o_v, o_e) = (2, 3)$: for each non-prime order $n \geq 9$.

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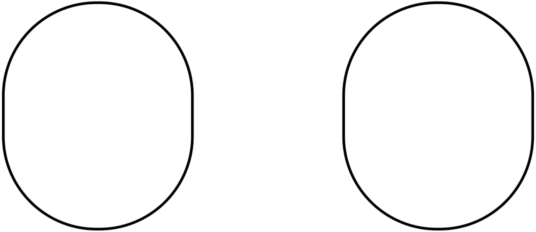

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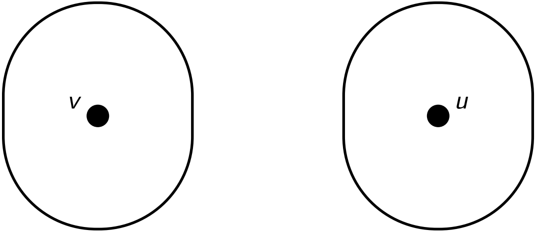

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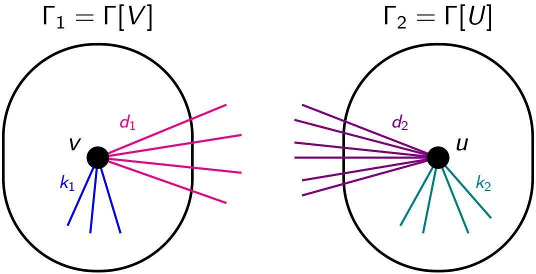

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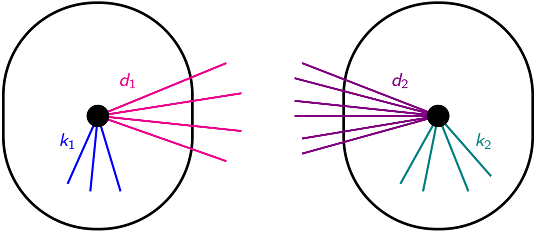

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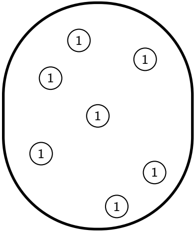

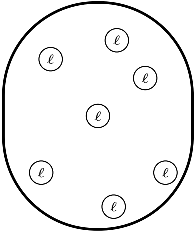

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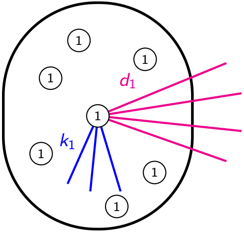

$k_1 + d_1 \ell = 0$

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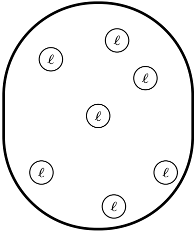

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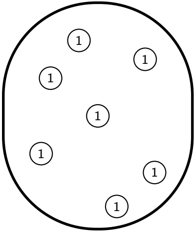

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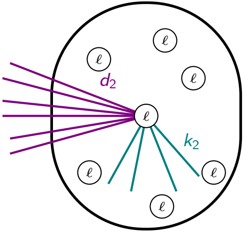

$\forall u \in U$: $\mathbf{x}(u) = \ell$

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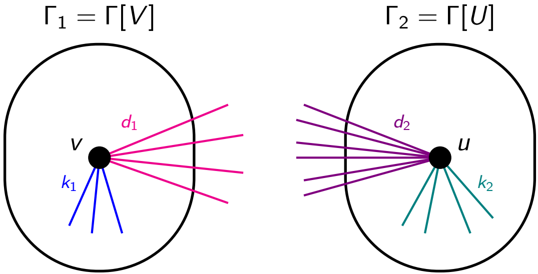

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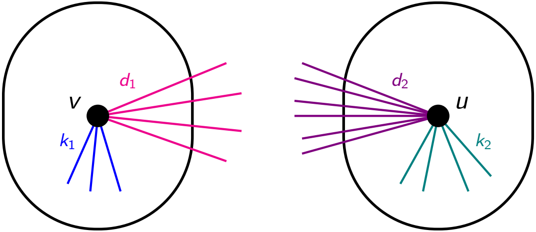
$n_1 = |V|$ $\qquad$ $n_2 = |U|$



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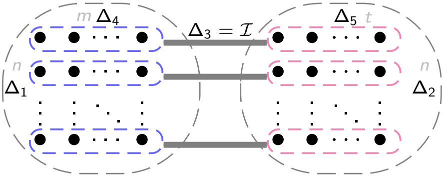
Since $n_1 d_1 = n_2 d_2$, we have that $\gcd(n_1, n_2) > 1$.



$\Gamma_1 = \mathcal{O}(\Delta_1) \times \mathcal{O}(\Delta_4)$ $\Gamma_2 = \mathcal{O}(\Delta_2) \times \mathcal{O}(\Delta_5)$

Proposition

Let $\Gamma = \Gamma(\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5)$. Then Γ admits a subgroup of the automorphism group having two vertex orbits and three edge orbits. Moreover, Γ is not a graph with $(o_v, o_e) = (2, 3)$ if and only if there exists an automorphism of Γ, mapping some vertex from Γ_1 to some vertex from Γ_2.

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Direct products of graphs

Let $\Gamma_1 = (V, E_1)$ and $\Gamma_2 = (U, E_2)$ be (multi)-graphs. A *direct product* $\Gamma_1 \times \Gamma_2$ is a graph with vertex-set $V \times U$ where two vertices (v_1, u_1), (v_2, u_2) are adjacent iff $v_1 \sim v_2$ and $u_1 \sim u_2$.

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Let Γ_1 and Γ_2 be (multi)graphs and let $\Gamma = \Gamma_1 \times \Gamma_2$.

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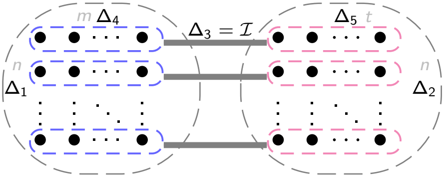
$$z_0 \left(\prod_{i=1}^{r} z_i\right)\left(\prod_{i=1}^{q} z_i'\right) \qquad \left(\prod_{i=1}^{r} d_i\right)\left(\prod_{i=1}^{q} (z_i' - 1)\right)$$

order valence



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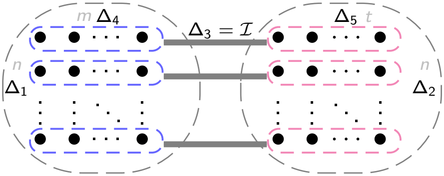

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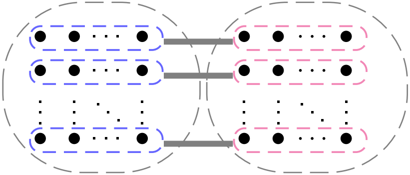

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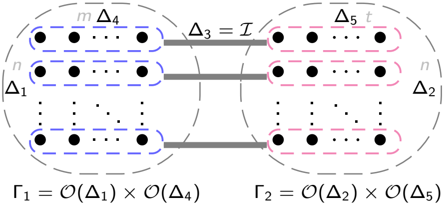

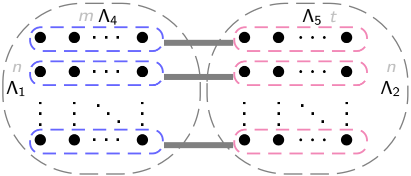

Non-singular, AT graphs $\Lambda_1, \Lambda_2, \Lambda_4, \Lambda_5$

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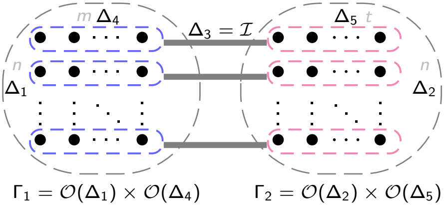

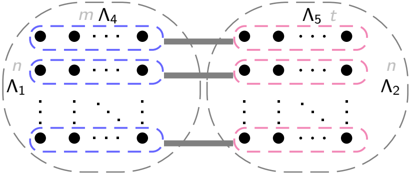

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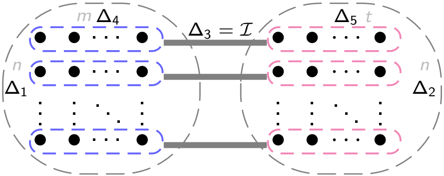

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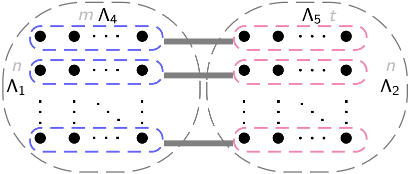

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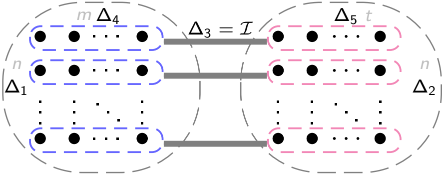

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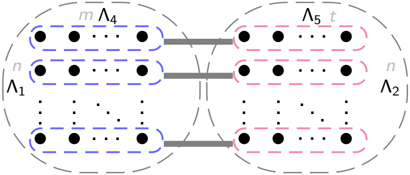

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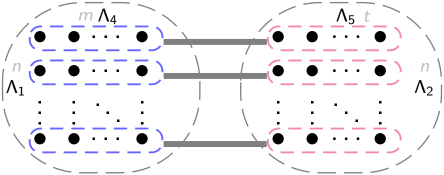
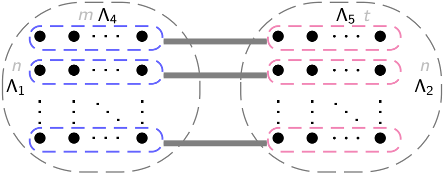
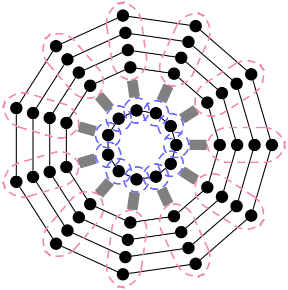
The first case: Λ_1 is of valence 2



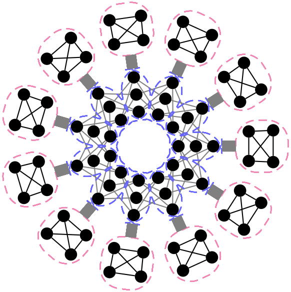
The first case: Λ_1 is of valence 2





$\Lambda_1 = C_{11}$
$\Lambda_2 = C_{11}$
$\Lambda_4 = K_1^o$
$\Lambda_5 = 4K_1^o$

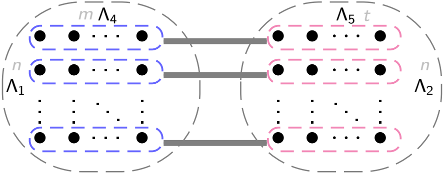
$2+4 \neq 2+1$
$2 \cdot 2 = 1 \cdot 4$



$\Lambda_1 = C_{11}$
$\Lambda_2 = 11K_1^o$
$\Lambda_4 = K_3$
$\Lambda_5 = K_4$

$4+4 \neq 3+3$
$4 \cdot 3 = 3 \cdot 4$

The first case: Λ_1 is of valence 2



Corollary

Let $n \geq 3$ be an odd integer. There exists a nut graph with $(o_v, o_e) = (2, 3)$ of order ns for each

$$s \in \{3, 5, 7, 11, 13, 29, 31, 37, 41, 47, 53, 59, 83, 101, 103, 109, 127, 131, 137, 139\}.$$

1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49
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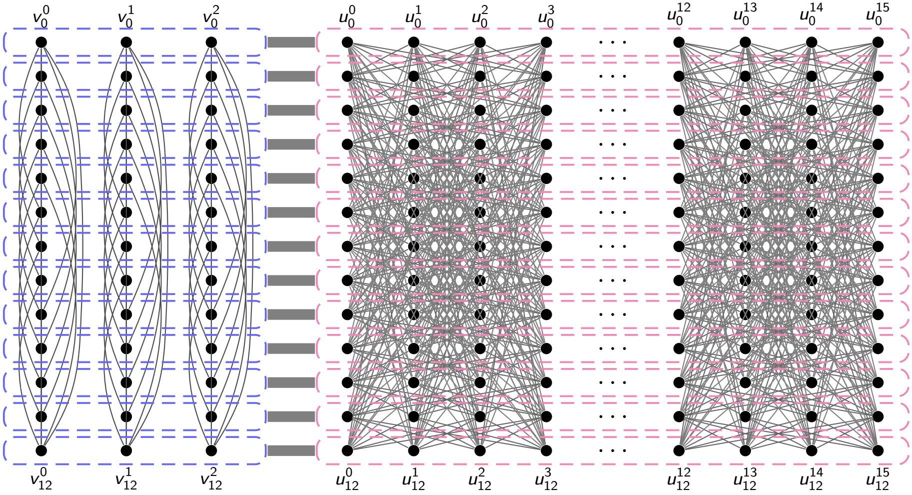
		1455	1457		1461	1463	1465	1467	1469		1473	1475	1477	1479			1485			1491		1495	1497	

The second case: Λ_1 of valence 4

$\Lambda_1 = \Lambda_2 = Cay(\mathbb{Z}_{13}; \{\pm 1, \pm 5\})$ $\Lambda_4 = 3K_1^o$ $\Lambda_5 = 4K_4$



$4 + 16 \neq 12 + 3$ $4 \cdot 12 = 3 \cdot 16$

The second case: Λ_1 of valence 4

Corollary

Let $n \geq 5$ be an odd integer such that either $n \equiv 1 \pmod 4$ or n is not a prime. Then there exists a nut graph with $(o_v, o_e) = (2, 3)$ of order ns for each $s \in \{17, 19, 23, 73, 79, 97, 107, 113\}$.

1				9			15			21		25	27			33	35		39			45		49
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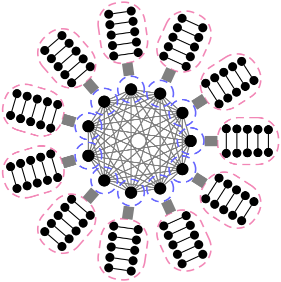
		1455	1457		1461	1463	1465	1467	1469		1473	1475	1477	1479			1485			1491		1495	1497	

The third case: Λ_1 of order p and even valence a, where $a \mid (p-1)$

$\Lambda_1 = K_{11}$ $\Lambda_2 = 11K_1^o$ $\Lambda_4 = K_1^o$ $\Lambda_5 = 5K_2$



$10 + 10 \neq 1 + 1$ $10 \cdot 1 = 1 \cdot 10$

1				9			15			21		25	27			33	35		39			45		49
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2001		2005	2007	2009		2013	2015		2019	2021	2023	2025			2031	2033	2035	2037		2041	2043	2045	2047	2049
2051		2055	2057	2059	2061		2065	2067		2071	2073	2075	2077	2079			2085			2091	2093	2095	2097	
2101	2103	2105	2107	2109			2115	2117	2119	2121	2123	2125	2127			2133	2135		2139			2145	2147	2149
2151		2155	2157	2159		2163	2165	2167	2169	2171	2173	2175	2177		2181	2183	2185	2187	2189	2191	2193	2195	2197	2199
2201		2205		2209	2211		2215	2217	2219		2223	2225	2227	2229	2231	2233	2235			2241		2245	2247	2249
	2253	2255	2257	2259	2261	2263	2265			2271		2275	2277	2279		2283	2285		2289	2291		2295		2299
2301	2303	2305	2307			2313	2315	2317	2319	2321	2323	2325	2327	2329	2331		2335	2337			2343	2345		2349
	2353	2355		2359	2361	2363	2365	2367	2369		2373	2375		2379			2385	2387		2391		2395	2397	
2401	2403	2405	2407	2409		2413	2415		2419	2421		2425	2427	2429	2431	2433	2435		2439		2443	2445		2449
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2501		2505	2507	2509	2511	2513	2515	2517	2519		2523	2525	2527	2529		2533	2535	2537		2541		2545	2547	
	2553	2555		2559	2561	2563	2565	2567	2569	2571	2573	2575	2577		2581	2583	2585	2587	2589			2595	2597	2599
2601	2603	2605	2607		2611	2613	2615		2619		2623	2625	2627	2629	2631		2635	2637	2639	2641	2643	2645		2649
2651	2653	2655			2661		2665	2667	2669		2673	2675		2679	2681		2685			2691		2695	2697	
2701	2703	2705		2709			2715	2717		2721	2723	2725	2727			2733	2735	2737	2739		2743	2745	2747	
2751		2755	2757	2759	2761	2763	2765		2769	2771	2773	2775		2779	2781	2783	2785	2787			2793	2795		2799
		2805	2807	2809	2811	2813	2815	2817		2821	2823	2825	2827	2829	2831		2835		2839	2841		2845	2847	2849
	2853	2855		2859		2863	2865	2867	2869	2871	2873	2875	2877		2881	2883	2885		2889	2891	2893	2895		2899
2901		2905	2907		2911	2913	2915		2919	2921	2923	2925		2929	2931	2933	2935	2937		2941	2943	2945	2947	2949
2951		2955		2959	2961		2965	2967			2973	2975	2977	2979	2981	2983	2985	2987	2989	2991	2993	2995	2997	

1501	1503	1505	1507	1509		1513	1515	1517	1519	1521		1525	1527	1529		1533	1535	1537	1539	1541		1545	1547	
1551		1555	1557		1561	1563	1565		1569		1573	1575	1577		1581		1585	1587	1589	1591	1593	1595		1599
	1603	1605			1611		1615	1617			1623	1625		1629	1631	1633	1635		1639	1641	1643	1645	1647	1649
1651	1653	1655		1659	1661		1665			1671	1673	1675	1677	1679	1681	1683	1685	1687	1689	1691		1695		
1701	1703	1705	1707		1711	1713	1715	1717	1719			1725	1727	1729	1731		1735	1737	1739		1743	1745		1749
1751		1755	1757		1761	1763	1765	1767	1769	1771	1773	1775		1779	1781		1785			1791	1793	1795	1797	1799
	1803	1805	1807	1809		1813	1815	1817	1819	1821		1825	1827	1829		1833	1835	1837	1839	1841	1843	1845		1849
1851	1853	1855	1857	1859		1863	1865		1869			1875			1881	1883	1885	1887		1891	1893	1895	1897	1899
	1903	1905		1909	1911		1915	1917	1919	1921	1923	1925	1927	1929			1935	1937	1939	1941	1943	1945	1947	
	1953	1955	1957	1959	1961	1963	1965	1967	1969	1971		1975	1977		1981	1983	1985		1989	1991		1995		
2001		2005	2007	2009		2013	2015		2019	2021	2023	2025			2031	2033	2035	2037		2041	2043	2045	2047	2049
2051		2055	2057	2059	2061		2065	2067		2071	2073	2075	2077	2079			2085			2091	2093	2095	2097	
2101	2103	2105	2107	2109			2115	2117	2119	2121	2123	2125	2127			2133	2135		2139			2145	2147	2149
2151		2155	2157	2159		2163	2165	2167	2169	2171	2173	2175	2177		2181	2183	2185	2187	2189	2191	2193	2195	2197	2199
2201		2205		2209	2211		2215	2217	2219		2223	2225	2227	2229	2231	2233	2235			2241		2245	2247	2249
	2253	2255	2257	2259	2261	2263	2265			2271		2275	2277	2279		2283	2285		2289	2291		2295		2299
2301	2303	2305	2307			2313	2315	2317	2319	2321	2323	2325	2327	2329	2331		2335	2337			2343	2345		2349
	2353	2355		2359	2361	2363	2365	2367	2369		2373	2375		2379			2385	2387		2391		2395	2397	
2401	2403	2405	2407	2409		2413	2415		2419	2421		2425	2427	2429	2431	2433	2435		2439		2443	2445		2449
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2501		2505	2507	2509	2511	2513	2515	2517	2519		2523	2525	2527	2529		2533	2535	2537		2541		2545	2547	
	2553	2555		2559	2561	2563	2565	2567	2569	2571	2573	2575	2577		2581	2583	2585	2587	2589			2595	2597	2599
2601	2603	2605	2607		2611	2613	2615		2619		2623	2625	2627	2629	2631		2635	2637	2639	2641	2643	2645		2649
2651	2653	2655			2661		2665	2667	2669		2673	2675		2679	2681		2685			2691		2695	2697	
2701	2703	2705		2709			2715	2717		2721	2723	2725	2727			2733	2735	2737	2739		2743	2745	2747	
2751		2755	2757	2759	2761	2763	2765		2769	2771	2773	2775		2779	2781	2783	2785	2787			2793	2795		2799
		2805	2807	2809	2811	2813	2815	2817		2821	2823	2825	2827	2829	2831		2835		2839	2841		2845	2847	2849
	2853	2855		2859		2863	2865	2867	2869	2871	2873	2875	2877		2881	2883	2885		2889	2891	2893	2895		2899
2901		2905	2907		2911	2913	2915		2919	2921	2923	2925		2929	2931	2933	2935	2937		2941	2943	2945	2947	2949
2951		2955		2959	2961		2965	2967			2973	2975	2977	2979	2981	2983	2985	2987	2989	2991	2993	2995	2997	

Computational data up to order one million

N	#odd non-prime int.	#not covered	%not covered
1 000	332	0	0
2 500	883	0	0
5 000	1 831	2	0.00109
100 000	40 408	46	0.00113
500 000	208 462	227	0.00108
600 000	250 902	267	0.00106
700 000	293 457	317	0.00108
800 000	336 049	364	0.00108
900 000	378 726	417	0.00110
1 000 000	421 502	457	0.00108

The first few remaining orders (up to 10 000):

2 839, 4 313, 5 377, 6 103, 9 101, 9 557

A note-worthy example

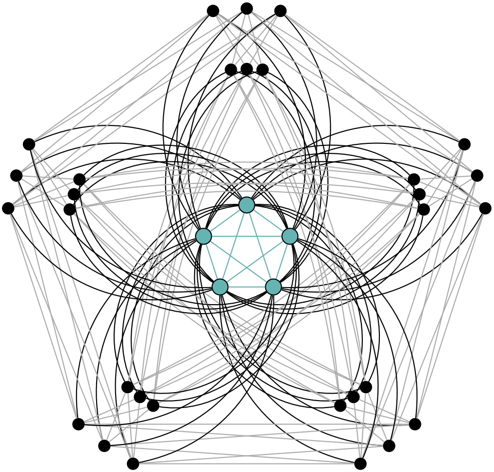

Figure: An example of a nut graph with the parameter 6-tuple $\langle 5, 4, 12, 30, 6, 2 \rangle$.

Thank you!